Math 601 – **Spring** 2014
Homework #5

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1. Prove that there is no orientation-reversing homeomorphism of $\mathbb{CP}^2$; that is, there is no homeomorphism $\varphi : \mathbb{CP}^2 \longrightarrow \mathbb{CP}^2$ such that $\varphi_*[\mathbb{CP}^2] = -[\mathbb{CP}^2]$.

2. Prove that there is an orientation-reversing homeomorphism of $S^2 \times S^2$.

3. (Hatcher, number 3.3.3) Show that every covering space f an orientable manifold is an orientable manifold.

4. (Hatcher, number 3.3.7) For a map $f : M \longrightarrow N$ between connected closed orientable n-manifolds, with fundamental classes $[M]$ and $[N]$, the *degree* of f is defined to be the integer d such that $f_*[M] = d[N]$, so the sign of the degree depends on the choice of fundamental classes. Show that for any connected closed orientable n-manifold M there is a degree 1 map $M \longrightarrow S^n$.

5. (Hatcher, number 3.3.16) Show that $(\alpha \cap \phi) \cap \psi = \alpha \cap (\phi \cup \psi)$ for all $\alpha \in C_k(X; R)$, $\phi \in C^l(X; R)$, and $\psi \in C^m(X; R)$. Deduce that cap product makes $H_*(X; R)$ a right $H^*(X; R)$-module.